COMBINED HEAT AND MASS TRANSFER FOR OPPOSING FILM FLOWS OF LIQUID AND GAS

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The combined heat and mass transfer for opposing film flows of liquid and gas is investigated numerically on the basis of solving the conjugate equations of convective mass and energy transfer.

Calculation of combined heat and mass transfer is very urgent for various thermophysical and chemical-engineering processes. This problem takes on particular importance for nonisothermal pellicular absorption. The literature lacks any theoretical analysis of this problem on the basis of the equations of combined heat and mass transfer.

In the present work, for the example of opposing pellicular two-phase flows in rectangular channels, the problem of calculating the heat and mass transfer is numerically solved and investigated in a conjugate formulation. It is assumed that the liquid and gas films move laminarly, the laws of velocity variation in the phases are known, the physicochemical properties of the medium are constant, the dissipative terms in the energy equations are so slight that they may be neglected, there are no source terms in the mass- and heat-transfer equations.

Under these assumptions, the system of equations of convective mass and energy transfer takes the form

$$u_i(y_i)\frac{\partial \tilde{C}_i}{\partial x_i} = D_i \frac{\partial^2 \tilde{C}_i}{\partial y_i^2}; \tag{1}$$

$$u_i(y_i)\frac{\partial \tilde{T}_i}{\partial x_i} = \varkappa_i \frac{\partial^2 \tilde{T}_i}{\partial y_i^2}, \text{ where } i = 1,\ 2, \tag{2}$$

with initial and boundary conditions: above, at the liquid inlet to the channel, that is

$$\text{when} \quad x_1 = 0,\ \tilde{C}_1 = \tilde{C}_{01};\quad \tilde{T}_1 = \tilde{T}_{01}; \tag{3}$$

below, at the gas inlet, that is

$$\text{when} \quad x_2 = 0,\ \tilde{C}_2 = \tilde{C}_{02};\quad \tilde{T}_2 = \tilde{T}_{02}; \tag{4}$$

at the channel wall

$$\text{when} \quad y_1 = 0,\ \frac{\partial \tilde{C}_1}{\partial y_1} = 0; \tag{5}$$

$$\tilde{T}_1 = T_w; \tag{6}$$

at the vertical axis of the channel

$$\text{when} \quad y_2 = 0,\ \frac{\partial \tilde{C}_2}{\partial y_2} = 0;\quad \frac{\partial \tilde{T}_2}{\partial y_2} = 0; \tag{7}$$

at the phase interface with $y_1 = h_0$ and $y_2 = R - h_0$, the condition of thermodynamic phase equilibrium holds

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$$\tilde{T}_1 = \tilde{T}_2 = \tilde{T}; \tag{8}$$

$$\tilde{C}_1 = k_0(\tilde{T})\,\tilde{C}_2 + k_1(\tilde{T}), \tag{9}$$

as well as the material and heat balances

$$D_1 \frac{\partial \tilde{C}_1}{\partial y_1} = -D_2 \frac{\partial \tilde{C}_2}{\partial y_2} = J_{\text{bo}}; \quad \lambda_1 \frac{\partial \tilde{T}_1}{\partial y_1} = -\lambda_2 \frac{\partial \tilde{T}_2}{\partial y_2} + J_{\text{bo}}\, r_A\,. \tag{10}$$

The system in Eqs. (1) and (2) with the initial and boundary conditions in Eqs. (3)-(10) describes the combined heat and mass transfer in two-phase laminar pellicular absorption in conditions of counterflow motion of the phases. The solution of this system allows the real concentration and temperature fields in the apparatus to be obtained, and its efficiency to be calculated on this basis.

The problem is investigated for a flow of moderate velocity, although the method of solution is applicable for any specified velocity profile.

The coefficients of the equilibrium dependence in Eq. (9) are specified as follows

$$k_0(\tilde{T}) = A; \quad k_1(\tilde{T}) = B\tilde{T} + G. \tag{11}$$

Taking account of Eq. (11), Eq. (9) reduces to the form

$$\tilde{C}_1 = A\tilde{C}_2 + B\,\tilde{T} + G. \tag{12}$$

Introducing dimensionless variables by means of the relations

$$\begin{gathered}
\tilde{C}_1 = \tilde{C}_{01} + A\left(\tilde{C}_{02} - \frac{\tilde{C}_{01} - B\tilde{T}_{02} - G}{A}\right) C_1; \quad \tilde{C}_2 = \frac{\tilde{C}_{01} - B\,\tilde{T}_{02} - G}{A} + \left(\tilde{C}_{02} - \frac{\tilde{C}_{01} - B\tilde{T}_{02} - G}{A}\right) C_2; \\
\tilde{T}_1 = \tilde{T}_{01} + (\tilde{T}_{02} - \tilde{T}_{01})\,T_1; \quad \tilde{T}_2 = (\tilde{T}_{02} - \tilde{T}_{01})\,T_2 + \tilde{T}_{01}; \\
u_1 = u_{01}\bar{u}_1; \quad u_2 = u_{02}\,\bar{u}_2; \\
y_1 = h_0\eta_1; \; y_2 = (R - h_0)\,\eta_2; \; x_1 = \frac{h_0^2\,u_{01}}{D_1}\,z_1; \quad x_2 = \frac{h_0^2 u_{01}}{D_1}\,z_2\,,
\end{gathered} \tag{13}$$

Eqs. (1) and (2) transform to the equations

$$\begin{gathered}
\bar{u}_1 \frac{\partial C_1}{\partial z_1} = \frac{\partial^2 C_1}{\partial \eta_1^2}\,; \quad \bar{u}_2\beta^2 \frac{\partial C_2}{\partial z_2} = \frac{\partial^2 C_2}{\partial \eta_2^2}\,; \\
\frac{\bar{u}_1}{\mathrm{Le}_1} \frac{\partial T_1}{\partial z_1} = \frac{\partial^2 T_1}{\partial \eta_1^2}\,; \bar{u}_2\beta_1^2 \frac{\partial T_2}{\partial z_2} = \frac{\partial^2 T_2}{\partial \eta_2^2}\,,
\end{gathered} \tag{14}$$

and the initial and boundary conditions in Eqs. (3)-(8), (10), and (12) take the form

$$\text{when } z_1 = 0 \quad C_1 = 0; \quad T_1 = 0; \text{when } z_2 = 0 \quad C_2 = 1; \quad T_2 = 1; \tag{15}$$

$$\text{when } \eta_1 = 0 \quad \frac{\partial C_1}{\partial \eta_1} = 0; \quad T_1 = T^*, \tag{16}$$

where

$$T^* = (T_W - \tilde{T}_{01})/(\tilde{T}_{02} - \tilde{T}_{01});$$

$$\text{when } \eta_2 = 0 \quad \partial C_2/\partial \eta_2 = 0; \quad \partial T_2/\partial \eta_2 = 0; \tag{17}$$

$$\text{when } \eta_1 = \eta_2 = 1 \quad T_1 = T_2; \quad C_1 = C_2 + B^*(T_2 - 1); \tag{18}$$

$$\frac{\partial C_2}{\partial \eta_2} + \beta^2\varepsilon \frac{\partial C_1}{\partial \eta_1} = 0; \quad \frac{\partial T_2}{\partial \eta_2} + \varphi_1 \frac{\partial T_1}{\partial \eta_1} = -\varphi_2 \frac{\partial C_2}{\partial \eta_2}\,, \tag{19}$$

where

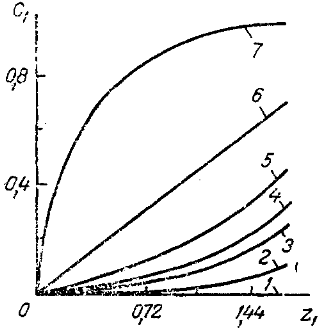

Fig. 1

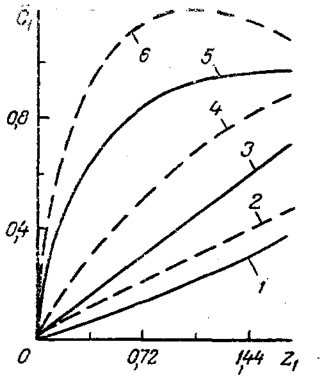

Fig. 2

Fig. 1. Dependence of the integral-mean concentration on the length of the contact device for $B^*=-0.005$; $\beta^2=1$; $\beta_1^2=0.4777$; $\varphi_1=782.382$; $\varphi_2=0$; $\mathrm{Le}_1=42.346$, with various values of ε: 1) ∞; 2) 10; 3) 4; 4) 3; 5) 2; 6) 1; 7) 0.

Fig. 2. Dependence of the integral-mean concentration on the length of the contact device with various values of β^2 and ε: 1, 2) $\beta^2 = 10^3$, $\varepsilon = 0.1$; 3, 4) 1, 1; 5, 6) 1, 0.01. The continuous curves correspond to $B^* = -0.005$ and the dashed curves to $B^* = -0.5$; values of φ_1, φ_2, Le_1 as in Fig. 1.

$$B^* = \frac{B(\tilde{T}_{02}-\tilde{T}_{01})}{A\tilde{C}_{02}-\tilde{C}_{01}+B\tilde{T}_{02}+G}; \tag{20}$$

$$\varphi_1 = \frac{\lambda_1}{\lambda_2}\left(\frac{R-h_0}{h_0}\right); \tag{21}$$

$$\varphi_2 = \frac{A\tilde{C}_{02}-\tilde{C}_{01}+B\tilde{T}_{02}+G}{A(\tilde{T}_{02}-\tilde{T}_{01})}\,\frac{D_2}{\lambda_2}\,r_A. \tag{22}$$

The system obtained in Eq. (14) with the initial and boundary conditions in Eqs. (15)-(19) is solved numerically by the method proposed in [1]. An iterative scheme is used; in each iteration, the boundary values of the variables are recalculated by means of Eqs. (18) and (19). The solutions are matched in the corresponding cross sections

$$i_1 = N - i_2,$$

where N is the number of cross sections over the height of the contact device; i_1 is the number of the cross section in the coordinate system (z_1, η_1) (counting downward); i_2 is the number of the cross section in the coordinate system (z_2, η_2) (counting upward).

On reaching the specific accuracy, the local concentration and temperature values in the phases are calculated, as well as the integral mean values in each cross section, according to the formulas

$$\begin{gathered}\bar{C}_1(z_1) = \int_0^1 C_1(z_1, \eta_1)\,d\eta_1;\quad \bar{T}_1(z_1) = \int_0^1 T_1(z_1, \eta_1)\,d\eta_1;\\ \bar{C}_2(z_2) = \int_0^1 C_2(z_2, \eta_2)\,d\eta_2;\quad \bar{T}_2(z_2) = \int_0^1 T_2(z_2, \eta_2)\,d\eta_2,\end{gathered} \tag{23}$$

and also the values of the boundary mass and heat fluxes, various kinetic characteristics of the process, etc.

Analysis of the results of solving the problem of two-phase conjugate heat and mass transfer shows that there is a certain analogy between this solution and the solution of the two-phase mass-transfer problem [2]. This analogy consists in the corresponding influence of

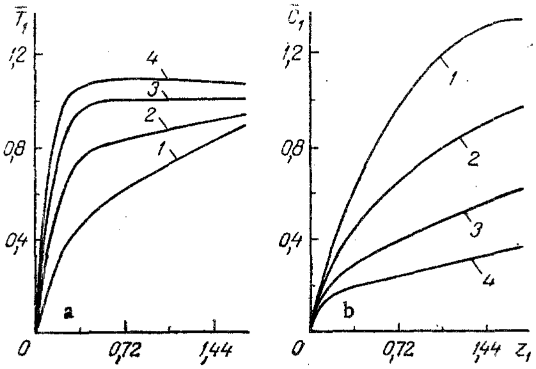

Fig. 3. Dependence of the integral mean temperature (a) and concentration (b) on the length of the contact device for $\beta^2 = 10^4$, $\varepsilon = 10^{-2}$, $B^* = -5$ at various φ_2: 1) 0; 2) 0.222; 3) 0.554; 4) 0.997; values of φ_1 and Le_1 as in Fig. 1.

the parameters β^2 and ε on the variation in characteristics of the process. Thus, if the solution of the problem is written as a function of the position of the point on the coordinate plane — $X = \log \varepsilon$, $Y = \log (1/\varepsilon\beta^2)$ — then it may be concluded that, in the upper left-hand corner of the plane (when $\varepsilon \leqslant 10^{-1}$ and $\varepsilon\beta^2 \leqslant 10^{-1}$), the two-phase problem degenerates to a single-phase problem, since in this case the resistance to heat and mass transfer is practically concentrated in the liquid phase. In the lower right-hand corner of the plane (when $\varepsilon \geqslant 10$ and $\varepsilon\beta^2 \geqslant 10$), the resistance is concentrated in the gas phase, which also leads to degeneration of the two-phase problem to a one-phase problem. In the lower left-hand and upper right-hand corners of the (X, Y) plane, the solution practically depends only on the parameter $\varepsilon\beta^2$. Therefore, before using the solutions of the equations of combined two-phase heat and mass transfer, it is necessary to determine the parameters ε and $\varepsilon\beta^2$, i.e., to find the corresponding point in the (X, Y) plane. Depending on the position of this point, either the single-phase (resistance to heat and mass transfer concentrated in one of the phases) or two-phase solution must be used.

Characteristic examples of the dependence of the dimensionless mean-integral concentrations in the liquid phase on the dimensionless length of the contact device are shown in Fig. 1, from which it is evident that, with decrease in ε (the quantity characterizing the volume of the liquid film) the resistance to mass transfer gradually shifts from the gas phase ($\varepsilon \to \infty$) to the liquid phase ($\varepsilon \to 0$).

In analyzing the solution of the problem of two-phase nonisothermal absorption, it is of special interest to investigate the temperature influence on the mass transfer. In imposing the boundary temperature condition in Eq. (6) (isothermal wall) on the initial system in Eqs. (1) and (2), this influence is slight. This corresponds completely to the physical picture of the process of combined heat and mass transfer, since the condition of constant wall temperature of the contact device with the liberation of additional heat of solution in the liquid phase — Eq. (10) — may only be maintained with heat loss through the wall of the apparatus. Therefore, the liquid-phase temperature remains practically constant and close to T_W and has essentially no influence on the mass transfer.

A different picture is seen when the boundary condition in Eq. (6) is replaced by a condition corresponding to adiabatic heat and mass transfer, that is

$$\frac{\partial \tilde{T}_1}{\partial y_1} = 0 \text{ when } y_1 = 0$$

or in dimensionless form

$$\frac{\partial T_1}{\partial \eta_1} = 0 \text{ when } \eta_1 = 0. \qquad (24)$$

This law of temperature variation at the wall of the apparatus means implies the possibility of heat accumulation in the liquid phase, increase in liquid-phase temperature, and

TABLE 1. Example of the Calculation of the Drying Process of Wet Air by Diethylene Glycol

Re_1	Re_2	Values at inlet				Values at outlet			
		$\tilde{T}_1$, °C	$\tilde{T}_2$, °C	$\tilde{C}_1$, mass %	$\tilde{C}_2$ g/m^3	$\tilde{T}_1$, °C	$\tilde{T}_2$, °C	$\tilde{C}_1$, mass %	$\tilde{C}_2$, g/m^3
5,272	2420	21,0	37,0	1,080	16,740	24,00	30,82	1,355	10,638
5,272	310	20,5	38,0	1,293	5,840	21,50	21,92	1,322	0,893
5,272	1142	21,0	39,0	1,293	4,483	22,19	27,04	1,339	2,308
5,272	310	21,0	39,0	1,453	5,504	21,73	22,32	1,479	1,041
4,393	698	23,0	38,5	1,444	6,529	24,80	27,60	1,503	2,773
4,393	310	22,5	38,5	1,653	6,537	23,86	23,90	1,688	1,571
4,393	698	25,0	41,0	1,744	7,916	27,00	29,79	1,811	3,641
4,393	1585	25,0	40,5	1,861	7,692	27,43	33,04	1,958	4,952

hence modification of the equilibrium and real values of the concentration in the phases.

Investigating the results of calculating nonisothermal absorption shows that thermal influence on the mass transfer appears basically as a function of the two parameters B^* and φ_2; see Eqs. (20) and (22). As follows from Eq. (18), the parameter B^* characterizes the temperature contribution to the equilibrium dependence. With increase in B^*, the influence of the boundary temperature on the surface concentrations of the liquid and gas phases increases.

Curves of the integral-mean concentrations in the liquid phase on the length of the contact device are shown in Fig. 2 for various values of β^2, ε, and B^*. Analysis of these curves leads to the conclusion that, with growth in the absolute value of B^*, the role of the liquid-phase resistance increases and that of the gas-phase component decreases.

Another parameter characterizing the influence of heat transfer on the mass transfer in absorption is the parameter φ_2, proportional to the heat of absorption liberated in the liquid phase; see Eq. (22). Increase in φ_2 leads to increase in heat flux in the liquid phase and, in the presence of the boundary condition in Eq. (24), to heating of the liquid film.

These two types of temperature influence on the concentration as a function of B^* and φ_2 are of different physical character. Therefore, simultaneous increase in φ_2 and the absolute value of B^* leads to perceptible change in the temperature profiles over the cross section and the length of the contact device. Curves of the integral-mean dimensionless temperature of the liquid phase as a function of the dimensionless length of the apparatus, which are independent interest in this case, are shown in Fig. 3a. The change in T_1, in turn, entails change in C_1, as shown in Fig. 3b (the values of φ_1, Le_1, and Le_2 remain unchanged).

As is evident from an analysis of the solution of the problem, the influence of the heat of absorption on the concentration dependence appears in the region where $\beta\sqrt[3]{\varepsilon} \geqslant 1$. If $\beta\sqrt[3]{\varepsilon} < 1$, however, the influence of the heat of absorption on the mass transfer may be neglected.

Table 1 gives the initial and calculated (from the given method) final parameters of the drying process of wet air by diethylene glycol in a film apparatus of height 0.87 m and channel width 0.018 m. The physicochemical data necessary for the calculations are taken from [3-5]: $D_1 = 0.3897 \cdot 10^{-9}$ m^2/sec; $D_2 = 2 \cdot 10^{-5}$ m^2/sec; $\lambda_1 = 0.205$ W/m·K; $\lambda_2 = 0.0255$ W/m·K; $\kappa_1 = 9.94 \cdot 10^{-8}$ m^2/sec; $\kappa_2 = 2.11 \cdot 10^{-5}$ m^2/sec.

NOTATION

x_i, y_i, $i = 1, 2$, orthogonal coordinate system (the axis x_1 is directed in the direction of motion of the liquid phase and the axis y_1 from the channel wall to its center; the axes x_2, y_2 run in the opposite direction to x_1 and y_1; h_0 is the film thickness; R is the channel halfwidth; $k_0(\tilde{T})$, $k_1(\tilde{T})$ are the phase-equilibrium coefficients; λ_i, κ_i, D_i, $i = 1, 2$ are the thermal conductivity, thermal diffusivity, and diffusion coefficient; $\tilde{C}_i$, $\tilde{T}_i$, $i = 1, 2$ are the concentration and temperature in the phases; $\tilde{C}_{01}$, $\tilde{T}_{01}$ are the concentration and temperature of the liquid film at the channel inlet; $\tilde{C}_{02}$, $\tilde{T}_{02}$ are the concentration and temperature of the gas at the channel inlet; u_{01}, u_{02} are the mean mass-flow velocities of the liquid and gas phases; $\beta^2 = \frac{u_{02}}{u_{01}} \frac{D_1}{D_2} \left(\frac{R - h_0}{h_0}\right)^2$; $\beta_1^2 = \beta^2/Le_2$; $\varepsilon = \frac{u_{01}}{u_{02}} \frac{h_0}{R - h_0}$; Le_1, Le_2 are the Lewis numbers for the liquid and gas phases; r_A is the heat of absorption; z_i, η_i, $i = 1, 2$, are the dimensionless coordinates. Subscripts: 1, liquid phase; 2, gas phase.

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EFFECT OF INITIAL CONDITIONS ON THE CHARACTERISTICS OF FREE JETS EXITING FROM CURVILINEAR CHANNELS

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Results are presented from an experimental study of the effect of centrifugal forces in a toroidal channel on the characteristics of a jet escaping therefrom.

The study of the principles of motion of free jets escaping from channels of various geometries is of great practical importance in many branches of industry and power production [1]. Study of the characteristics of such jet flows contributes to the development of that little-studied area of hydrodynamics concerned with mixing of turbulent jets with a nonsymmetric initial velocity and static pressure distribution. It should be noted that technological applications require calculation of the cutoff angle, i.e., the spatial position of the channel output section plane. Such a problem was considered as long ago as the 1930s in the theory of steam turbines. However, the unique features of escape of a jet from toroidal channels with an oblique mouth have yet to be sufficiently studied.

In the present study special attentiaon will be given to clarification of the effect of centrifugal forces upon the characteristics of a jet existing a toroidal curvilinear channel (Fig. 1c). The internal diameter of the channel d = 12 mm. The ratio between radius of curvature of the channel axis R_ω and the diameter d is: $R_\omega = 2.914d$. The mouth was oriented at an angle of 45° to the center of curvature. The entrance to the curvilinear segment was preceded by a cylindrical channel 50d in length. For comparison, jet flows from a cylindrical channel with straight (Fig. 1a) and oblique (45° angle) (Fig. 1b) mouths were also studied. Figure 2 shows shadow photographs of a jet of cooled air with initial velocity 10 m/sec. An IZK-463 viewer with 800-mm field of view permitted visual observation of jet propagation at quite significant distances (50 diameters). The visualizing knife was installed both along and across the jet. Exposure time for photography was $2 \cdot 10^{-3}$ sec.

It is well known that initial conditions at the nozzle mouth exert a definite effect on flow within a jet [2]. Initial boundary layers or an initial nonuniformity of the velocity profile, as a rule, is accompanied by an increased level of turbulence and turbulent viscosity, which leads to a more intense mixing of the jet as compared to the case of a uniform initial velocity profile. It is known [3] that the maximum value of turbulent viscosity in a turbulent boundary layer is determined by the dynamic viscosity u* and the thickness of the boundary layer δ:

$$E_{\max} \simeq 0{,}07u^*\delta, \tag{1}$$

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